J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

SYJC PRELIUM - 02

SECTION - I

Q1. Attempt ANY SIX of the following (12)

01. Find X and Y if $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$; $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

SOLUTION

$$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \qquad X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \qquad X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \qquad 2Y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \qquad Y = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} \qquad Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

02. Find dy/dx if $y = \cos^{-1}\left(2x\sqrt{1 - x^2}\right)$

SOLUTION

$y = \cos^{-1} 2x\sqrt{1 - x^2}$ Put $x = \sin\theta$

$y = \cos^{-1}\left(2\sin\theta\sqrt{1 - \sin^2\theta}\right)$

$y = \cos^{-1}\left(2\sin\theta\sqrt{\cos^2\theta}\right)$

$y = \cos^{-1}(2\sin\theta\cos\theta)$

$y = \cos^{-1}(\sin 2\theta)$

$y = \cos^{-1}\cos(\pi/2 - 2\theta)$

$y = \pi/2 - 2\theta$

$y = \pi/2 - 2\sin^{-1}x$

$\frac{dy}{dx} = -\frac{2`}{\sqrt{1 - x^2}}$

04. Find dy/dx if $x = \sin^3\theta$, $y = \cos^3\theta$

SOLUTION

$x = \sin^3\theta$

$\dfrac{dx}{d\theta} = 3\sin^2\theta \dfrac{d}{d\theta}\sin\theta$

$= 3\sin^2\theta .\cos\theta$

$y = \cos^3\theta$

$\dfrac{dy}{d\theta} = 3\cos^2\theta \dfrac{d}{d\theta}\cos\theta$

$= -3\cos^2\theta .\sin\theta$

$\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}$

$= \dfrac{-3\cos^2\theta .\sin\theta}{3\sin^2\theta .\cos\theta}$

$= -\cot\theta$

04. Write negations of the following statements

1. $\forall\ y \in N$, $y^2 + 3 \le 7$

Negation : $\exists\ y \in N$, such that $y^2 + 3 > 7$

2. if the lines are parallel then their slopes are equal

Using : $\sim(P \to Q) \equiv P \wedge \sim Q$

Negation : lines are parallel and their slopes are not equal

05. find elasticity of demand if the marginal revenue is Rs 50 and the price is Rs 75

SOLUTION

$$R_m = R_A\left(1 - \frac{1}{\eta}\right)$$

$$50 = 75\left(1 - \frac{1}{\eta}\right)$$

$$\frac{50}{75} = 1 - \frac{1}{\eta}$$

$$\frac{2}{3} = 1 - \frac{1}{\eta}$$

$$\frac{1}{\eta} = 1 - \frac{2}{3}$$

$$\frac{1}{\eta} = \frac{1}{3} \qquad \eta = 3$$

06. State which of the following sentences are statements . In case of statement , write down the truth value

a) Every quadratic equation has only real roots

ans : the given sentence is a logical statement . Truth value : F

b) $\sqrt{-4}$ is a rational number

ans : the given sentence is a logical statement . Truth value : F

07. Evaluate : $\int \frac{\sec^2 x}{\tan^2 x + 4} dx$

SOLUTION

PUT $\tan x = t$

$\sec^2 x . dx = dt$

THE SUM IS

$$= \int \frac{1}{t^2 + 4} dt$$

$$= \int \frac{1}{t^2 + 2^2}. dt$$

$$= \frac{1}{a} \tan^{-1} \frac{t}{a} + c$$

$$= \frac{1}{2} \tan^{-1} \frac{t}{2} + c$$

Resubs.

$$= \frac{1}{2} \tan^{-1} \left(\frac{\tan x}{2}\right) + c$$

08. if $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then find $|AB|$

SOLUTION

AB

$$= \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 3 & 2 + 4 \\ 2 + 6 & 4 + 8 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 \\ 8 & 12 \end{bmatrix}$$

$|AB| = 4(12) - 8(6) = 48 - 48 = 0$

Q2.(A) Attempt ANY TWO of the following (06)

Q2A

01. if the function given below is continuous at x = 2 and x = 4 then find a & b

$$f(x) = \begin{cases} x^2 + ax + b & ; \quad x < 2 \\ 3x + 2 & ; \quad 2 \le x \le 4 \\ 2ax + 5b & ; \quad 4 < x \end{cases}$$

SOLUTION

PART – 1

STEP 1

$\lim_{x\to 2^-} f(x)$

$= \lim_{x\to 2} x^2 + ax + b$

$= 2^2 + a(2) + b$

$= 4 + 2a + b$

STEP 2

$\lim_{x\to 2^+} f(x)$

$= \lim_{x\to 2} 3x + 2$

$= 3(2) + 2 = 8$

STEP 3

$f(2) = 3(2) + 2 = 8$

STEP 4

Since the f is continuous at x = 2

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$4 + 2a + b = 8 = 8$

$2a + b = 4$ ………………. (1)

PART – 2

STEP 1

$\lim_{x\to 4^-} f(x)$

$= \lim_{x\to 4} 3x + 2$

$= 3(4) + 2 = 14$

STEP 2

$\lim_{x\to 4^+} f(x)$

$= \lim_{x\to 4} 2ax + 5b$

$= 2a(4) + 5b$

$= 8a + 5b$

STEP 3

$f(4) = 3(4) + 2 = 14$

STEP 4

Since the f is continuous at x = 4

$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = f(4)$

$14 = 8a + 5b = 14$

$8a + 5b = 14$ ………………. (2)

Solving (1) and (2) : a = 3 , b = –2

02.

Using rules of negations , write the negation of the following

a) $p \wedge (q \to r)$ b) $\sim p \vee \sim q$

a) $\sim [p \wedge (q \to r)]$

$\sim p \vee \sim (q \to r)$ …. De Morgan's Law

$\sim p \vee (q \wedge \sim r)$ … $\sim(P \to Q) \equiv P \wedge \sim Q$

b) $\sim \ \sim p \vee \sim q$

$\sim(\sim p) \wedge \sim(\sim q)$ …. De Morgan's Law

$p \wedge q$

03.

a manufacturing company produces x items at the total cost of (180 + 4x). The demand function is p = 240 − x . Find x for which the profit is increasing

SOLUTION

$R = px$

$= 240x - x^2$

$C = 180 + 4x$

$\pi = R - C$

$= 240x - x^2 - 180 - 4x$

$= 236x - x^2 - 180$

For Profit increasing

$$\frac{d\pi}{dx} > 0$$

$236 - 2x > 0$

$236 > 2x$

$118 > x$

$x < 118$

(B) Attempt ANY TWO of the following (08)

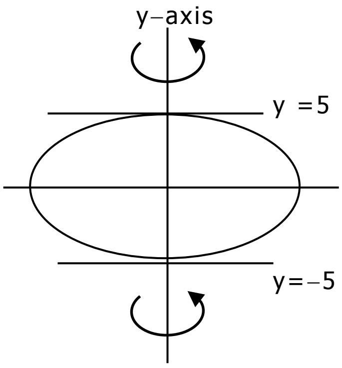
Q2B

Q1. Find the volume of the solid obtained by the complete revolution of the ellipse

$$\frac{x^2}{36} + \frac{y^2}{25} = 1 \text{ about } y - \text{axis}$$

SOLUTION

STEP 1 :



$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a^2 = 36$; $a = 6$

$b^2 = 25$, $b = 5$

STEP 2 :

$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

$$\frac{x^2}{36} = 1 - \frac{y^2}{25}$$

$$\frac{x^2}{36} = \frac{25 - y^2}{25}$$

$$x^2 = \frac{36}{25}(25 - y^2)$$

STEP 3 :

$$V = \pi \int_{-5}^{5} x^2 . dy$$

About y – axis

$$= \pi \int_{-5}^{5} \frac{36}{25}(25 - y^2).dy$$

$$= \frac{36\pi}{25} \int_{-5}^{5} (25 - y^2).dy$$

$$= \frac{36\pi}{25} \left[25y - \frac{y^3}{3}\right]_{-5}^{5}$$

$$= \frac{36\pi}{25} \left\{\left[125 - \frac{125}{3}\right] - \left[-125 + \frac{125}{3}\right]\right\}$$

$$= \frac{36\pi}{25} \left\{\left[\frac{375 - 125}{3}\right] - \left[\frac{-375 + 125}{3}\right]\right\}$$

$$= \frac{36\pi}{25} \left\{\left[\frac{250}{3}\right] - \left[\frac{-250}{3}\right]\right\}$$

$$= \frac{36\pi}{25} \left[\frac{500}{3}\right]$$

$= 240\ \pi$ cubic units

02. Evaluate : $\int \tan^{-1}\sqrt{x}\, dx$

SOLUTION

LET $\sqrt{x} = t$

$$\frac{1}{2\sqrt{x}}dx = dt$$

$$dx = 2\sqrt{x}dt$$

$$dx = 2t\ dt$$

$$= \int \tan^{-1}t \ .\ 2t\ dt$$

$$= 2\int \tan^{-1}t\ .\ t\ dt$$

$$= 2\left(\tan^{-1}t\ .\int t\ dt \quad - \int \frac{d}{dt}\tan^{-1}t \int t dt\ dt\right)$$

$$= 2\left(\tan^{-1}t\ .\ \frac{t^2}{2} \quad - \int \frac{1}{1+t^2}\ \frac{t^2}{2}\ dt\right)$$

$$= 2\left(\frac{t^2}{2}\ \tan^{-1}t\ . \ - \ \frac{1}{2}\int \frac{t^2}{1+t^2}\ dt\right)$$

$$= \quad t^2\ \tan^{-1}t\ . \ - \int \frac{1+t^2-1}{1+t^2}\ dt$$

$$= \quad t^2\ \tan^{-1}t\ . \ - \int 1 - \frac{1}{1+t^2}\ dt$$

$$= \quad t^2\ \tan^{-1}t\ . \ - \ t \ + \ \tan^{-1}t \ + c$$

RESUBSTITUTE

$$= \quad x.\tan^{-1}\sqrt{x}\ . \ - \ \sqrt{x} \ + \ \tan^{-1}\sqrt{x} \ + c$$

Q2B

03.

the processing cost of x bags is $\frac{2x^3}{3} - 48x^2$,

and packing & dispatching cost is (1289x + 3750) Find the number of bags to be manufactured so as to minimize the marginal cost . Also find the marginal cost for that number of bags

SOLUTION

$$C = \frac{2x^3}{3} - 48x^2 + 1289x + 3750$$

$$C_M = \frac{dC}{dx}$$

$$= \frac{6x^2}{3} - 96x + 1289$$

$$= 2x^2 - 96x + 1289$$

$$\frac{dC_M}{dx} = 4x - 96$$

$$\frac{d^2C_M}{dx^2} = 4$$

$$\frac{dC_M}{dx} = 0$$

$4x - 96 = 0 \quad x = 24$

$$\left.\frac{d^2C_M}{dx^2}\right|_{x\,=\,24} = 4 > 0$$

C_M is minimum at x = 24

$$\left.C_M\right|_{x\,=\,24} = 2(24)^2 - 96(24) + 1289$$

$$= 2(576) - 2304 + 1289$$

$$= 1152 - 2304 + 1289$$

$$= 137$$

Q3.(A) Attempt ANY TWO of the following (06)

01.

Using ALGEBRA OF STATEMENTS , prove

$$p \wedge [(\sim p \vee q) \vee \sim q] \equiv p$$

SOLUTION

$p \wedge [(\sim p \vee q) \vee \sim q]$

$\equiv p \wedge [\sim p \vee (q \vee \sim q)]$ ... ASSOCIATIVE LAW

$\equiv p \wedge (\sim p \vee t)$ ... COMPLEMENT LAW

$\equiv p \wedge t$ ... IDENTITY LAW

$\equiv p$

02. $$\int_2^3 \frac{x}{(x+2)(x+3)}\,dx$$

SOLUTION

$$\frac{x}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$x = A(x+3) + B(x+2)$$

Put $x = -3$

$-3 = B(-3+2)$

$-3 = B(-1)$

$3 = B$

Put $x = -2$

$-2 = A(-2+3)$

$-2 = A(1)$

$-2 = A$

HENCE

$$\frac{x}{(x+2)(x+3)} = \frac{-2}{x+2} + \frac{3}{x+3}$$

BACK IN THE SUM

$$= \int_2^3 \left(\frac{-2}{x+2} + \frac{3}{x+3}\right) dx$$

$$= \left[-2\log|x+2| + 3\log|x+3|\right]_2^3$$

$$= [-2\log 5 + 3\log 6] - [-2\log 4 + 3\log 5]$$

$$= -2\log 5 + 3\log 6 + 2\log 4 - 3\log 5$$

$$= 2\log 4 + 3\log 6 - 5\log 5$$

Q3A

$$= \log 4^2 + \log 6^3 - \log 4^5$$

$$= \log 16 + \log 216 - \log 3125$$

$$= \log\left(\frac{16 \times 216}{3125}\right)$$

$$= \log\left(\frac{3456}{3125}\right)$$

03. if $\sin y = x.\sin(5+y)$;

prove that $\dfrac{dy}{dx} = \dfrac{\sin^2(5+y)}{\sin 5}$

SOLUTION

$$x = \frac{\sin y}{\sin(5+y)}$$

$$\frac{dx}{dy} = \frac{\sin(5+y)\frac{d}{dy}\sin y - \sin y\frac{d}{dy}\sin(5+y)}{\sin^2(5+y)}$$

$$= \frac{\sin(5+y).\cos y - \sin y\cos(5+y)\,{}^{d}/_{dy}(5+y)}{\sin^2(5+y)}$$

$$= \frac{\sin(5+y).\cos y - \cos(5+y).\sin y}{\sin^2(5+y)}$$

$$= \frac{\sin(5+y-y)}{\sin^2(5+y)}$$

$$\frac{dx}{dy} = \frac{\sin 5}{\sin^2(5+y)}$$

$$\frac{dy}{dx} = \frac{1}{dx/dy} = \frac{\sin^2(5+y)}{\sin 5}$$ ……… PROVED

(B) Attempt ANY TWO of the following

Q3B (08)

$$I = \int_0^2 x^2(2-x)^{1/2}\,dx$$

SOLUTION

USING $\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$

$$I = \int_0^2 (2-x)^2 \,.\, x^{1/2}\,dx$$

$$I = \int_0^2 (4-4x+x^2).\,x^{1/2}\,dx$$

$$I = \int_0^2 (4x^{1/2} - 4x^{3/2} + x^{5/2})\,dx$$

$$I = \left[\frac{4x^{3/2}}{\frac{3}{2}} - \frac{4x^{5/2}}{\frac{5}{2}} + \frac{x^{7/2}}{\frac{7}{2}}\right]_0^2$$

$$I = \left[\frac{8}{3}x^{3/2} - \frac{8}{5}x^{5/2} + \frac{2x^{7/2}}{7}\right]_0^2$$

$$I = \frac{8}{3}2^{3/2} - \frac{8}{5}2^{5/2} + \frac{2}{7}2^{7/2}$$

$$I = \frac{8}{3}2\sqrt{2} - \frac{8}{5}2^2\sqrt{2} + \frac{2}{7}2^3\sqrt{2}$$

$$I = \frac{16\sqrt{2}}{3} - \frac{32\sqrt{2}}{5} + \frac{16\sqrt{2}}{7}$$

$$I = 16\sqrt{2}\left(\frac{1}{3} - \frac{2}{5} + \frac{1}{7}\right)$$

$$I = 16\sqrt{2}\,\frac{35 - 42 + 15}{105}$$

$$I = 16\sqrt{2}\,\frac{8}{105}$$

$$I = \frac{128\sqrt{2}}{105}$$

02.

if f is continuous at $x = 0$, then find f(0) where

$$f(x) = \frac{(3^{\sin x} - 1)^2}{x\,.\,\log(1+x)} \quad ; \quad x \neq 0$$

SOLUTION

$$\lim_{x\to 0} f(x)$$

$$\lim_{x\to 0} \frac{(3^{\sin x} - 1)^2}{x\,.\,\log(1+x)}$$

Divide N and D by $\sin^2 x$, $\sin^2 x \neq 0$

$$\lim_{x\to 0} \frac{\frac{(3^{\sin x} - 1)^2}{\sin^2 x} \quad \sin^2 x}{x\,.\,\log(1+x)}$$

Divide N and D by x^2 , $x^2 \neq 0$

$$\lim_{x\to 0} \frac{\frac{(3^{\sin x} - 1)^2}{\sin^2 x} \quad \frac{\sin^2 x}{x^2}}{\frac{x\,.\,\log(1+x)}{x^2}}$$

$$\lim_{x\to 0} \frac{\left(\frac{3^{\sin x} - 1}{\sin x}\right)^2 \left(\frac{\sin x}{x}\right)^2}{\frac{\log(1+x)}{x}}$$

$$= \frac{(\log 3)^2 \,.\, (1)2}{1}$$

$$= (\log 3)^2$$

Since f(x) is continuous at $x = 0$

$$f(0) = \lim_{x\to 0} f(x)$$

$$= (\log 3)^2$$

03. $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix}$

Q3B

Verify : A.(adj A) = (adj A).A = |A|.I

SOLUTION

$A_{11} = (-1)^{1+1} \begin{vmatrix} 0 & -2 \\ 0 & 2 \end{vmatrix} = 1(0 - 0) = 0$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -1(9 + 2) = -11$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 1(0 - 0) = 0$

$A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -1(-3 - 0) = 3$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 1(3 - 2) = 1$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -1(0 + 1) = -1$

$A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 1(2 - 0) = 2$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -1(-2 - 6) = 8$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = 1(0 + 3) = 3$

COFACTOR MATRIX OF A

$$\begin{pmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{pmatrix}$$

ADJ A = TRANSPOSE OF THE COFACTOR MATRIX

$$= \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix}$$

|A|

$= 1(0 + 0) + 1(9 + 2) + 2(0 - 0)$

$= 11$

LHS 1

= A.(adj A)

$$= \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix} \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 + 11 + 0 & 3 - 1 - 2 & 2 - 8 + 6 \\ 0 - 0 - 0 & 9 + 0 + 2 & 6 + 0 - 6 \\ 0 - 0 + 0 & 3 + 0 - 3 & 2 + 0 + 9 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$

LHS 2

= (adj A) . A

$$= \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 + 9 + 2 & 0 + 0 + 0 & 0 - 6 + 6 \\ -11 + 3 + 8 & 11 + 0 + 0 & -22 - 2 + 24 \\ 0 - 3 + 3 & 0 - 0 + 0 & 0 + 2 + 9 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$

RHS

= |A|.I

$$= 11 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$

HENCE A.(adj A) = (adj A).A = |A|.I

SECTION - II

Q1. Attempt ANY SIX of the following (12)

Q4

01. Find correlation coefficient between x and y for the following data

$n = 100$, $\bar{x} = 62$, $\bar{y} = 53$, $\sigma x = 10$, $\sigma y = 12$, $\Sigma(x - \bar{x})(y - \bar{y}) = 8000$

SOLUTION

$$r = \frac{cov(x,y)}{\sigma x \cdot \sigma y}$$

$$= \frac{\frac{\Sigma(x-\bar{x})(y-\bar{y})}{n}}{\sigma x \cdot \sigma y}$$

$$= \frac{\frac{8000}{100}}{10.12}$$

$$= \frac{80}{10.12}$$

$$= \frac{2}{3}$$

02. a building is insured for 80% of its value . The annual premium at 70 paise percent amounts to ₹ 2,800 . Fire damaged the building to the extent of 60% of its value . How much amount for damage can be claimed under the policy

SOLUTION

Property value = ₹ x

Insured value $= \frac{80x}{100} = \frac{4x}{5}$

Rate of premium = 70 paise percent

= 0.70%

Premium = ₹ 2800

$2800 = \frac{0.70}{100} \times \frac{4x}{5}$

$2800 = \frac{7}{1000} \times \frac{4x}{5}$

$2800 = \frac{28x}{5000}$

$x = 100 \times 5000$

$x = 5,00,000$

Property value = ₹ 5,00,000

Loss $= \frac{60}{100} \times 5,00,000$

= ₹ 3,00,000

Claim = 80% of loss

$= \frac{80}{100} \times 3,00,000$

= ₹ 2,40,000

03. The coefficient of rank correlation for a certain group of data is 0.5 . If $\sum d^2$ = 42 , assuming no ranks are repeated ; find the no. of pairs of observation

SOLUTION

$R = 0.5$; $\Sigma d^2 = 42$

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$0.5 = 1 - \frac{6(42)}{n(n^2 - 1)}$$

$$\frac{6(42)}{n(n^2 - 1)} = 1 - 0.5$$

$$\frac{6(42)}{n(n^2 - 1)} = 0.5$$

$$\frac{6(42)}{n(n^2 - 1)} = \frac{1}{2}$$

$$n(n^2 - 1) = 6 \times 42 \times 2$$

$$n(n^2 - 1) = 3 \times 2 \times 3 \times 2 \times 7 \times 2$$

$$(n - 1).n.(n + 1) = 7 \times 8 \times 9$$

On comparing , n = 8

Q4

04.

Maya and Jaya started a business by investing equal amount . After 8 months Jaya withdrew her amount and Priya entered the business with same amount of capital . At the end of the year there was a profit of ₹ 13,200 . Find their share of profit

SOLUTION

STEP 1 :

Profits will be shared in the
'RATIO OF PERIOD OF INVESTMENT'

	MAYA		JAYA		PRIYA
=	12	:	8	:	4

TOTAL = 24

STEP 2 :

PROFIT = ₹ 13,200

Maya's share of profit $= \frac{12}{24} \times 13,200$

= ₹ 6,600

Jaya's share of profit $= \frac{8}{24} \times 13,200$

= ₹ 4,400

Priya'sshare of profit $= \frac{4}{24} \times 13,200$

= ₹ 2,200

05. Calculate CDR for district A and B and compare

SOLUTION

Age Group (Years)	DISTRICT A		DISTRICT B	
	NO. OF PERSONS **P**	NO. OF DEATHS **D**	NO. OF PERSONS **P**	NO. OF DEATHS **D**
0 – 10	1000	18	3000	70
10 – 55	3000	32	7000	50
Above 55	2000	41	1000	24
	ΣP = 6000	**ΣD = 91**	**ΣP = 11000**	**ΣD = 144**

$$CDR(A) = \frac{\Sigma D}{\Sigma P} \times 1000 = \frac{91}{6000} \times 1000 = 15.17$$

(DEATHS PER THOUSAND)

$$CDR(B) = \frac{\Sigma D}{\Sigma P} \times 1000 = \frac{144}{11000} \times 1000 = 13.09$$

(DEATHS PER THOUSAND)

COMMENT : CDR(B) < CDR(A) . HENCE DISTRICT B IS HEALTHIER THAN DISTRICT A

06. the probability of defective bolts in a workshop is 40% . Find the mean and variance of defective bolts out os 10 bolts

SOLUTION

$n = 10$,

p = probability of defective bolt $= \frac{40}{100} = \frac{2}{5}$

$q = 1 - p = \frac{3}{5}$

$$X \sim B(10, {}^2/_5)$$

$$\text{Mean} = np = 10 \times \frac{2}{5} = 4$$

$$\text{Variance} = npq = 10 \times \frac{2}{5} \times \frac{3}{5} = 2.4$$

07. The ratio of incomes of Salim & Javed was 20:11 . Three years later income of Salim has increased by 20% and income of Javed was increased by Rs 500 . Now the ratio of their incomes become 3 : 2 . Find original incomes of Salim and Javed

SOLUTION

Let income of Salim = 20x

Income of Javed = 11x

As per the given condition

$$\frac{20x + \frac{20(20x)}{100}}{11x + 500} = \frac{3}{2}$$

$$\frac{20x + 4x}{11x + 500} = \frac{3}{2}$$

$$\frac{24x}{11x + 500} = \frac{3}{2}$$

$$48x = 33x + 1500$$

$$x = 100$$

$\therefore$

Salim's original income = 20(100) = ₹ 2000

Javed's original income = 11(100) = ₹ 1100

08. for an immediate annuity paid for 3 years with interest compounded at 10% p.a. its present value is Rs 10,000 . What is the accumulated value after 3 years ($1.1^3 = 1.331$)

SOLUTION

$$A = P(1 + i)^n$$

$$= 10000(1 + 0.1)^3$$

$$= 10000(1.1)^3$$

$$= 10000(1.331)$$

$$= ₹\ 13{,}310$$

Q5.(A) Attempt ANY TWO of the following (06)

Q5A

01. Obtain the expected value and variance of a random variable X for the following probability distribution

x	−2	−1	0	1	2	3
P(X = x)	0.1	k	0.2	2k	0.3	k

STEP 1: $\Sigma p(x) = 1$

$0.1 + k + 0.2 + 2k + 0.3 + k = 1$

$4k + 0.6 = 1$

$4k = 0.4 \qquad k = 0.1$

STEP 2:

x	−2	−1	0	1	2	3	
p(x)	0.1	0.1	0.2	0.2	0.3	0.1	
pi.xi	−0.2	− 0.1	0	0.2	0.6	0.3	Σpi.xi = 0.8
pi.xi^2	0.4	0.1	0	0.2	1.2	0.9	Σpi.xi^2 = 2.8

STEP 3 : $E(x) = \Sigma pi.xi = 0.8$

STEP 4 : $Var(x) = \Sigma pi.xi^2 - E(x)^2 = 2.8 - 0.8^2 = 2.8 - 0.64 = 2.16$

02. Calculate the Spearman's rank Correlation coefficient between the following marks given by two judges to 8 contestants in the election elocution

Marks by A :	81	72	60	33	29	11	56	42
Marks by B :	75	56	42	15	30	20	60	80

SOLUTION

A	B	x	y	d = |x − y|	d^2
81	75	1	2	1	1
72	56	2	4	2	4
60	42	3	5	2	4
33	15	6	8	2	4
29	30	7	6	1	1
11	20	8	7	1	1
56	60	4	3	1	1
42	80	5	1	4	16
					$\Sigma d^2 = 32$

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(32)}{8(64 - 1)}$$

$$= 1 - \frac{6(32)}{8(63)}$$

$$= 1 - \frac{8}{21}$$

$$= \frac{13}{21}$$

$$= 0.62$$

03. a wholesaler allows 25% trade discount and 5% cash discount . Find the list price of an article if it was sold for the net amount of Rs 1140 .

Q5A

SOLUTION

List Price	= ₹	100
Less 25% T.D.	–	25
Invoice Price	= ₹	75
Less 5% C.D.	–	3.75
Net Selling Price	= ₹	71.25

Now When ;

Net SP = 71.25 ; List Price = 100

Net SP = ☐ 1140 ; List Price $= \frac{1140 \times 100}{71.25}$

= ₹ 1600

(B) Attempt ANY TWO of the following (08)

01. Find the sequence that minimizes total elapsed time (in hours) required to complete the following jobs on three machines M_1 , M_2 and M_3 in the order $M_1 M_2 M_3$. Also find the minimum elapsed time and idle time for all three machines

Q5B

Job	A	B	C	D	E
M_1	5	7	6	9	5
M_2	2	1	4	5	3
M_3	3	7	5	6	7

STEP 1 : Min time on M_1 = 5 ; Max time on M_2 = 5 ; Min time on M_3 = 3

Min (M_1) ≥ Max (M_2) ……… condition satisfied to convert 3 m/c's to 2 m/c's

STEP 2 : CONVERTING TO 2 FICTITIOUS M/C'S G & H

G = M_1 + M_2 , H = M_2 + M_3

Job	A	B	C	D	E
G	7	8	10	14	8
H	5	8	9	11	10

STEP 3 : OPTIMAL SEQUENCE

Min time **=** 5 on job A on machine H .

				A

Next min time =8 on job B & E on machine G .

B	**E**			**A**

Next min time = 9 on job C on machine H .

B	E		C	A

OPTIMAL SEQUENCE

B	E	D	C	A

STEP 4 **:** WORK TABLE

Job	B	E	D	C	A		total process time
M_1	7	5	9	6	5	=	32 hrs
M_2	1	3	5	4	2	=	15 hrs
M_3	7	7	6	5	3	=	28 hrs

JOBS	**M_1**		**IDLE**	**M_2**		**IDLE**	**M_3**		**IDLE**
	IN	**OUT**	**TIME**	**IN**	**OUT**	**TIME**	**IN**	**OUT**	**TIME**
						7			8
B	0	7	--	7	8	4	8	15	--
E	7	12	--	12	15	6	15	22	4
D	12	21	--	21	26	1	26	32	--
C	21	27	--	27	31	1	32	37	--
A	27	32	8	32	34	6	37	40	--

STEP 5 **:** Total elapsed time T = 40 hrs

Idle time on M_1 = T − (sum of processing time of all 5 jobs on M_1)

= 40 − 32

= 8 hrs

Idle time on M_2 = T − (sum of processing time of all 5 jobs on M_2)

= 40 − 15

= 25 hrs (CHECK − 7 + 4 + 6 + 1 + 1 + 6 = 25)

Idle time on M_3 = T − (sum of processing time of all 5 jobs on M_3)

= 40 − 28

= 12 hrs (CHECK − 8 + 4 = 12)

02. X : 6 2 10 4 8

Y : 9 11 b 8 7

Q5B

Arithmetic means of X and Y series are 6 and 8 respectively . Calculate correlation coefficient

SOLUTION

$$\bar{y} = \frac{\Sigma y}{n} \qquad 8 = \frac{9 + 11 + b + 8 + 7}{5} \rightarrow \quad 40 = 35 + b \qquad \therefore\ b = 5$$

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
6	9	0	1	0	1	0
2	11	−4	3	16	9	−12
10	5	4	−3	16	9	−12
4	8	−2	0	4	0	0
8	7	2	−1	4	1	−2
30 Σx	40 Σy	0 $\Sigma(x-\bar{x})$	0 $\Sigma(y-\bar{y})$	40 $\Sigma(x-\bar{x})^2$	20 $\Sigma(y-\bar{y})^2$	−26 $\Sigma(x-\bar{x})(y-\bar{y})$
$\bar{x} = 6$	$\bar{y} = 8$					

$$r = \frac{\Sigma(x-\bar{x}).(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}.$$

$$r = \frac{-26}{\sqrt{40} \times \sqrt{20}}$$

$$r = \frac{-26}{\sqrt{40 \times 20}}$$

$$r' = \frac{26}{\sqrt{40 \times 20}}$$

taking log on both sides

$$\log r' = \log 26 - \frac{1}{2}\left(\log 40 + \log 20\right)$$

$$\log r' = 1.4150 - \frac{1}{2}[\ 1.6021 + 1.3010\ ]$$

$$\log r' = 1.4150 - \frac{1}{2}(2.9031)$$

$$\log r' = 1.4150 - 1.4516$$

$$\log r' = \bar{1}.9634$$

$$r' = AL(\bar{1}.9634) = 0.9191$$

$$r = -0.9191$$

03.

Q5B

A bill drawn on 3rd June for 6 months was discounted on 17th Oct at rate of 5% . If the cash value is ₹ 43,500 , find the face value

SOLUTION

due 6 months @ 5 % p.a.

3rd Jun	17th Oct	6th Dec
	₹ 43,500	₹ x

STEP 1 :

Date of drawing	=	3 / 06
Add period of bill	+	6 months
Nominal due date	=	3 / 12
Add Grace days	+	3 days
Legal due date	=	6 / 12 ……. 6th December

STEP 2 :

Unexpired period

= 17th Oct – 6th Dec

	OCT		NOV		DEC
=	14	+	30	+	6

= 50 days

STEP 3 :

B.D. = F.V. – C.V.

= x – 43,500

STEP 4 :

B.D. = Int on F.V. for 50 days @ 5% p.a.

$$x - 43500 = \text{'x'} \times \frac{50}{365} \times \frac{5}{100}$$

$$x - 43500 = \frac{x}{146}$$

$$\frac{145x}{146} = 43500$$

$$x = \frac{43500 \times 146}{145}$$

$$x = ₹\ 43,800$$

Q6.(A) Attempt ANY TWO of the following (06)

01.

Age x	0	1	2
lx	1000	880	876
Tx	…..	…..	3323

Calculate e_0^0 , e_1^0 , e_2^0

SOLUTION

$$L_x = \frac{l_x + l_{x+1}}{2}$$

✓ $L_0 = \dfrac{l_0 + l_1}{2} = \dfrac{1000 + 880}{2} = 940$

✓ $L_1 = \dfrac{l_1 + l_2}{2} = \dfrac{880 + 876}{2} = 878$

$$T_{x+1} = T_x - L_x$$

✓ $T_2 = T_1 - L_1$

$3323 = T_1 - 878$

$T_1 = 4201$

✓ $T_1 = T_0 - L_0$

$4201 = T_0 - 940$

$T_0 = 5141$

$$e_x^0 = \frac{T_x}{l_x}$$

$e_0^0 = \dfrac{T_0}{l_0} = \dfrac{5141}{1000} = 5.141$

$e_1^0 = \dfrac{T_1}{l_1} = \dfrac{4201}{880} = 4.774$ (USE LOG)

$e_2^0 = \dfrac{T_2}{l_2} = \dfrac{3323}{876} = 3.793$ (USE LOG)

02. Suppose X is a random variable with pdf

$$f(x) = \frac{c}{x} \quad ; \quad 1 < x < 3$$

Find c ; E(X)

i) $\displaystyle\int_1^3 \frac{c}{x}\, dx = 1$

$\displaystyle c\int_1^3 \frac{1}{x}\, dx = 1$

$c\left[\log x\right]_1^3 = 1$

$c\left[\log 3 - \log 1\right] = 1$

$c \log 3 = 1$

$c = \dfrac{1}{\log 3}$

Q6A

Hence X is a r.v. with pdf

$$f(x) = \frac{1}{x.\log 3} \quad ; \quad 1 < x < 3$$

ii) $\displaystyle E(x) = \int_1^3 x.f(x)\, dx$

$\displaystyle = \int_1^3 x.\frac{1}{x.\log 3}\, dx$

$\displaystyle = \int_1^3 \frac{1}{\log 3}\, dx$

$\displaystyle = \left[\frac{x}{\log 3}\right]_1^3$

$= \dfrac{2}{\log 3}$

03. the equation of the line of regression of Y on X is $3x + 2y = 26$ and X on Y is $6x + y = 31$. Find var(X) if var(Y) = 36

Q6A

SOLUTION

Y on X : $3x + 2y = 26$

$$2y = -3x + 26$$

$$y = \frac{-3x}{2} + \frac{26}{2}$$

$$byx = -\frac{3}{2}$$

X on Y : $6x + y = 31$

$$6x = -y + 31$$

$$x = -\frac{1}{6}y + \frac{31}{6}$$

$$bxy = -\frac{1}{6}$$

$$r^2 = byx \times bxy$$

$$= -\frac{3}{2} \times -\frac{1}{6}$$

$$= \frac{1}{4}$$

$r = -\frac{1}{2}$ …… byx & bxy are negative

$$bxy = r\frac{\sigma x}{\sigma y}$$

$$\frac{-1}{6} = \frac{-1}{2}\frac{\sigma y}{6}$$

$$\sigma y = 2$$

$\therefore$ var(y)= 4

(B) Attempt ANY TWO of the following (08)

01. a team of 4 horses and 4 riders has entered the jumping show contest . The number of penalty points to be expected when each rider rides horse is shown below . How should the horses be assigned to the riders so as to minimize the expected loss . Also find the minimum expected loss

Q6B

RIDERS / HORSES

	H_1	H_2	H_3	H_4
R_1	12	3	3	2
R_2	1	11	4	13
R_3	11	10	6	11
R_4	5	8	1	7

SOLUTION

Reducing the matrix using 'ROW MINIMUM'

10	1	1	0
0	10	3	12
5	4	0	5
4	7	0	6

Reducing the matrix using 'COLUMN MINIMUM'

10	0	1	0
0	9	3	12
5	3	0	5
4	6	0	6

Allocation using 'SINGLE ZERO ROW COLUMN' method

allocation INCOMPLETE

10	$\boxed{0}$	1	$\cancel{0}$
$\boxed{0}$	9	3	12
5	3	$\boxed{0}$	5
4	6	$\cancel{0}$	6

REVISE THE MATRIX

STEP 1 – Drawing minimum lines to cover ALL '0's

	10	$\boxed{0}$	1	$\cancel{0}$
	$\boxed{0}$	9	3	12
√	5	3	$\boxed{0}$	5
√	4	6	$\cancel{0}$	6
			√	

STEP 2 – REVISE THE MATRIX
reduce all the uncovered elements by its minimum & add the same at the intersection

10	0	4	0
0	9	3	12
2	0	0	2
1	3	0	3

Re – allocation

10	$\cancel{0}$	4	$\boxed{0}$
$\boxed{0}$	9	3	12
2	$\boxed{0}$	$\cancel{0}$	2
1	3	$\boxed{0}$	3

Since every row and every column contains an ASSIGNED ZERO , ASSIGNMNET PROBLEM is SOLVED

OPTIMAL ASSIGNMENT : $R_1 - H_4$; $R_2 - H_1$; $R_3 - H_2$; $R_4 - H_3$

Minimum Penalty points = 2 + 1 + 10 + 1 = 14

02. Information on vehicles (in thousands) passing through seven different highways during a day (X) and number of accidents reported (Y) is given as

$\sum x = 105$; $\sum y = 409$; $\sum x^2 = 1681$; $\sum y^2 = 39350$; $\sum xy = 8075$.

Obtain linear regression of Y on X

SOLUTION

Q6B

$$\bar{x} = \frac{\sum x}{n} = \frac{105}{7} = 15$$

$$\bar{y} = \frac{\sum y}{n} = \frac{409}{7} = 58.43$$

$$byx = \frac{n\sum xy - \sum x.\sum y}{n\sum x^2 - (\sum x)^2}$$

$$= \frac{7(8075) - (105)(409)}{7(1681) - (105)^2}$$

$$= \frac{56525 - 42945}{11767 - 11025}$$

$$= \frac{13580}{742}$$

$$= 18.30$$

LOG CALC

4.1329
− 2.8704

AL 1.2625

18.30

Equation

$y - \bar{y} = byx\,(x - \bar{x})$

$y - 58.43 = 18.30(x - 15)$

$y - 58.43 = 18.30x - 274.5$

$y = 18.30x - 274.50 + 58.43$

$y = 18.30x - 216.07$

03. Minimize $z = 2x + y$

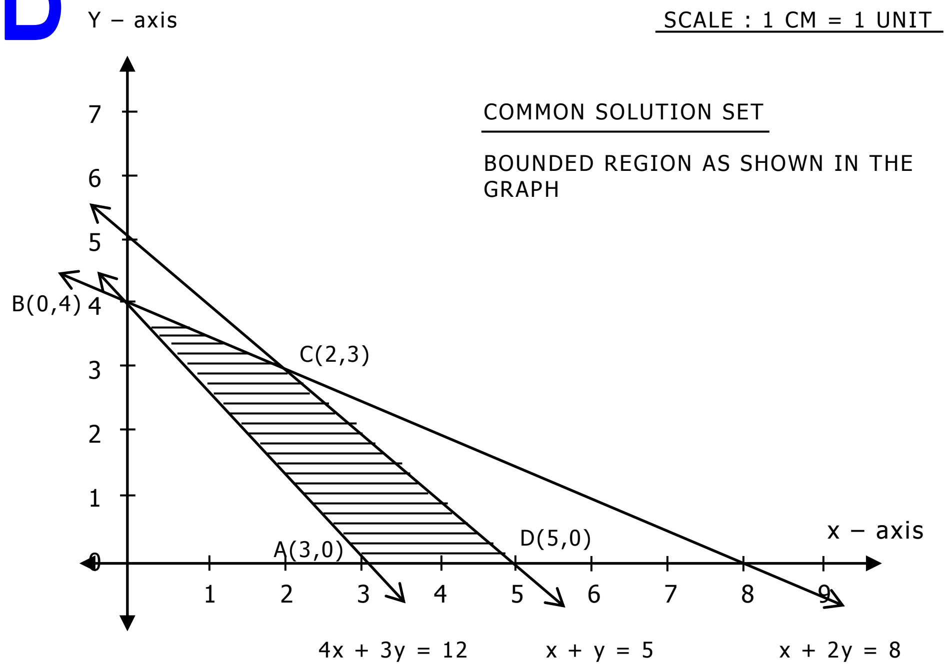
subject to : $x + y \le 5$, $x + 2y \le 8$, $4x + 3y \ge 12$, $x , y \ge 0$

Q6B

STEP 1 :

$x + y \le 5$	$x + y = 5$ cuts x − axis at (5,0) cuts y − axis at (0,5)	Put (0,0) in $x + y \le 5$ $0 \le 5$ SS : ORIGIN SIDE
$x + 2y \le 8$	$x + 2y = 8$ cuts x − axis at (8,0) cuts y − axis at (0,4)	Put (0,0) in $x + 2y \le 8$ $0 \le 8$ SS : ORIGIN SIDE
$4x + 3y \ge 12$	$4x + 3y = 12$ cuts x − axis at (3,0) cuts y − axis at (0,4)	Put (0,0) in $4x + 3y \ge 12$ $0 \ge 12$ (NOT SATISFIED) SS :NON-ORIGIN SIDE
$x , y \ge 0$		SS : I QUADRANT

STEP 2 :



STEP 3 :

CORNERS	$Z = 2x + y$	
A(3,0)	$Z = 2(3) + 0$	$= 6$
B(0,4)	$Z = 2(0) + 4$	$= 4$
C(2,3)	$Z = 2(2) + 3$	$= 7$
D(5,0)	$Z = 2(5) + 0$	$= 10$

STEP 4 :

Optimal Solution : $Z_{min} = 4$ at (0,4)